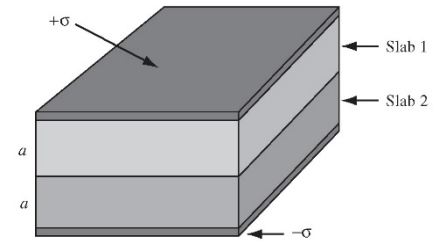


• $\frac{1}{r} = \frac{1}{R} \sum_{n=0}^{\infty} \left(\frac{r}{R}\right)^n P_n(\cos \vartheta) \quad r \leq R$

• $\int_{-1}^1 P_\ell(x) P_{\ell'}(x) dx = \int_0^\pi P_\ell(\cos \theta) P_{\ell'}(\cos \theta) \sin \theta d\theta = \begin{cases} 0 & \text{if } \ell' \neq \ell \\ \frac{2}{2\ell+1}, & \text{if } \ell' = \ell \end{cases}$

1. The space between the planes of a parallel-plate capacitor is filled with two slabs of linear dielectric material. Each slab has thickness a , so the total distance between the plates is $2a$. Slab 1 has a dielectric constant of 9, and slab 2 has a dielectric constant of 4. The free charge density on the top plate is σ and on the bottom plate $-\sigma$.

- (a) Find the electric displacement $\mathbf{D}$ in each slab. (5%)
 (b) Find the polarization $\mathbf{P}$ in each slab. (5%)
 (c) Find the potential difference between the metal plates. (5%)
 (d) Find the location and amount of all bound charges (ρ_b and σ_b). (5%)



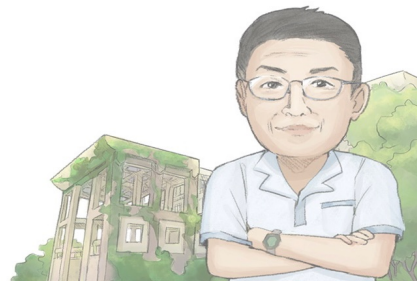
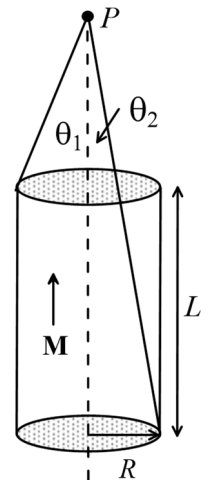
2. A sphere of radius R carries a polarization $\mathbf{P}(\mathbf{r}) = P\hat{\mathbf{z}}$, where P is a constant and $\hat{\mathbf{z}}$ is the unit vector.

- (a) Calculate the bound charges ρ_b and σ_b . (10%)
 (b) Find the electric potential and field inside the sphere. (10%)

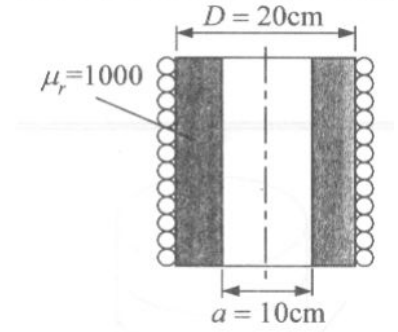
[Hint: $V = \frac{1}{4\pi\epsilon_0} \oint_S \frac{\sigma_b}{r} da' + \frac{1}{4\pi\epsilon_0} \int_v \frac{\rho_b}{r} d\tau'$]

3. A bar magnet of radius R and length L is magnetized with a uniform magnetization $\mathbf{M}$ in the z axis as shown in the figure.

- (a) Find the bound volume current $\mathbf{J}_b$ inside the magnet as well as the bound surface currents $\mathbf{K}_b$ on both ends and the cylindrical surface. (10%)
 (b) Find the magnetic field along the z axis. Use the technique similar to that of the solenoid and express your answer in terms of θ_1 and θ_2 . (10%)

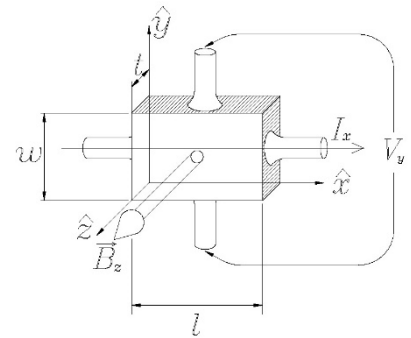


4. An *infinite* long magnetic material tube ($\mu_r = 1000$) of inner diameter $a = 10$ cm and outer diameter $D = 20$ cm is tightly wrapped with thin solenoid of 30 turns per unit cm, as shown in the figure. The current per turn I is 1 A. [Hint: $\mu_0 = 4\pi \times 10^{-7}$ H/m]



- Find the auxiliary field $\mathbf{H}$ at the following three regions $0 \leq r \leq a/2$, $a/2 \leq r \leq D/2$, and $r \geq D/2$. (7%)
- Find the magnetic field $\mathbf{B}$ at the following three regions $0 \leq r \leq a/2$, $a/2 \leq r \leq D/2$, and $r \geq D/2$. (7%)
- Explain why the magnetic field $\mathbf{B}$ is discontinuous at the boundary $r = a/2$. (6%) [Hint: Use the boundary condition for $\mathbf{B}$ field.]

5. Consider a conducting slab as shown below with length l in the x direction, width w in the y direction and thickness t in the z direction. The conductor has charge carrier of charge q and charge carrier drift velocity v_x when a current I_x flows in the positive x direction. The conductor is placed in a magnetic field perpendicular to the plane of the slab $\mathbf{B} = B_z \hat{z}$.



- When steady state is reached, there will be no net flow of charge in the y direction. Find the relation between E_y , B_z , and v_x . (7%)
- Find the resulting potential difference V_y (the **Hall voltage**) between the top and bottom of the slab, in terms of B_z , v_x , and the relevant dimensions of the slab. (7%)
- How do you determine the sign of the mobile charge carriers in a material? (6%) [Hint: n denotes the number of carriers per unit volume]

6. Boundary Conditions

- Write down the normal boundary condition $E^\perp$ and the tangential boundary conditions $E^\parallel$. [Hint: $\nabla \cdot \mathbf{E} = \rho/\epsilon_0$ and $\nabla \times \mathbf{E} = 0$] (5%)
- Write down the normal boundary condition $D^\perp$ and the tangential boundary conditions $D^\parallel$. [Hint: $\nabla \cdot \mathbf{D} = \rho_f$ and $\nabla \times \mathbf{D} = \nabla \times \mathbf{P}$] (5%)
- Write down the normal boundary condition $B^\perp$ and the tangential boundary conditions $B^\parallel$. [Hint: $\nabla \cdot \mathbf{B} = 0$ and $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$] (5%)
- Write down the normal boundary condition $H^\perp$ and the tangential boundary conditions $H^\parallel$. [Hint: $\nabla \cdot \mathbf{H} = -\nabla \cdot \mathbf{M}$ and $\nabla \times \mathbf{H} = \mathbf{J}_f$] (5%)

